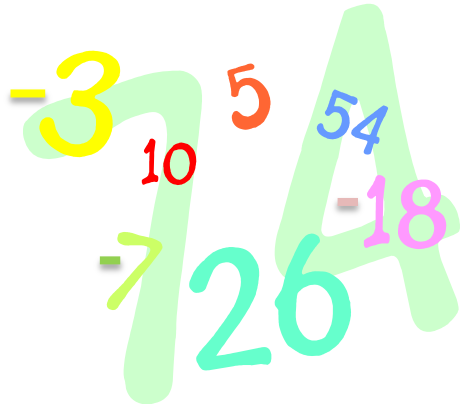
Lesson One: "*Properties of Exponents*"

Integers: Positive and Negative Whole Numbers

An integer is a number with no fractional part. Integers include the positive and the negative counting numbers.

Here are some words associated with integers:



A **negative number** is less than zero.
- 100, -25, -12 and -4 are all negative numbers.

A **positive number** is more than zero.
4, 12, 89 and 568 are all positive numbers.

The **negative sign** goes in front of a negative number.
Negative 4 is the same as – 4.

Integer Exponents: Positive and Negative Powers

Positive Exponents

An exponent tells us how many times to multiply the base number.

$3^5 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 243$

In this example, our base is 3, and our exponent is 5. This tells us to multiply the number three, by itself, five times.

REMEMBER: Exponents only expand what they directly touch!

Note:

Multiplication is repeated addition, so: $\mathbf{2 + 2 + 2 + 2 = 2 \cdot 4 = 8}$

Exponents are repeated *multiplication*, so: $\mathbf{2 \cdot 2 \cdot 2 \cdot 2 = 2^4 = 16}$

Negative Exponents

A negative exponent is not what it seems…

$2^{-3} = ?$

It is not possible to expand our base of *2*, negative *3* times as we would with positive exponents. Negative exponents mean we have an extra job to do before we can expand them.

$2^{-3} = \frac{1}{2^3} = ?$

The negative sign tell us that the number (with its exponent) is in the wrong place…**They need to move.** Consider a number with a negative exponent as being unhappy where he is. He who has the negative exponent is in the wrong/opposite location. To fix this, imagine there is a top and bottom (upstairs and downstairs), and once you move the number to its opposite location it will be happy.

$2^{-3} = \frac{1}{2^3} = \frac{1}{2 \cdot 2 \cdot 2} = \frac{1}{8}$

How to handle…simplify powers of zero.

Anything raised to the power of zero equals one…as long as the "0" power (exponent) touches all parts.

Example:

1. $3^0 = 1$
2. $-3^0 = -1$
3. $(-3)^0 = 1$
4. $(54304309^{2323})^0 = 1$

TEACHER NOTE: *Have students use their calculator to solve problem 4 for themselves.*

Properties of Exponents

Addition & Subtraction Rule:

TEACHER NOTE: *Before explaining the exponent rules, use the expansion method to justify the rules. Be sure to provide students with an opportunity to see the pattern and discover the rules for themselves*.

In order to add or subtract numbers with exponents, you must first find the value of each power, then add the two numbers.

For example, to add $3^3 + 4^2$, you must expand the exponents to get…

$$3^3 + 4^2 = (3 \cdot 3 \cdot 3) + (4 \cdot 4) = 27 + 16 = 43.$$

Product Rule:

1. When you multiply like bases, you keep the base the same and add the exponents.

   For example:

   $$5^2 \cdot 5^7 = 5^{(2+7)} = 5^9$$

2. To multiply exponential numbers raised to the same exponent, raise their product to that exponent.

   For Example:

   $$4^3 \cdot 5^3 = (4 \cdot 5)^3 = 20^3$$

3. If you are multiplying unlike bases you must be extra careful. You cannot simply add the exponents and/or multiply the bases. Instead, you must first find the value of each power, then add the two numbers

   For Example:

   $$7^2 \cdot 2^3 = (7 \cdot 7)(2 \cdot 2 \cdot 2) = 49 \cdot 8$$

Quotient Rule:

1. To divide exponential numbers raised to the same exponent, raise their quotient to that exponent.

   For example:

$$\frac{2^2}{3^2} = \left(\frac{2}{3}\right)^2 = \frac{2}{3} \cdot \frac{2}{3} = \frac{4}{9}$$

2. To divide two same-base exponential numbers or terms with different exponents, just subtract the exponents.

   For example:

$$\frac{3^6}{3^2} = 3^{(6-2)} = 3^4$$

NOTE: If you need to multiply or divide two exponential numbers that don't have the same base or exponent, you'll just have to do your work the old-fashioned way: multiply the exponential numbers out, and multiply or divide them accordingly

Power Rule:

1. When we encounter a power raised to another power, we multiply the exponents.

   For example:

$$(4^2)^3 = 4^{(2*3)} = 4^6$$

2. In some cases we must combine the power rule and negative exponents. In this case, distribute the power to each item inside <u>or</u> use pushdown method if the whole item is a negative

   Example 1 (Distribution first):

$$(3^2)^{-2} = 3^{(2*-2)} = 3^{-4} = \frac{1}{3^4}$$

   Example 2 ("Pushdown first):

$$(3^2)^{-2} = \frac{1}{(3^2)^2} = \frac{1}{3^{(2*2)}} = \frac{1}{3^4}$$